

Sgr A* as probe of the theory of supermassive compact objects without event horizon

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Abstract. In the present paper some consequences of the hypothesis that the supermassive compact object in the Galaxy centre relates to a class of objects without event horizon are examined. The possibility of the existence of such objects was substantiated by the author earlier. It is shown that accretion of a surrounding gas can cause nuclear combustion in the surface layer which, as a result of comptonization of the superincumbent hotter layer, may give a contribution to the observed Sgr A* radiation in the range $10^{15} \div 10^{20} \text{ Hz}$. It is found a contribution of the possible proper magnetic moment of the object to the observed synchrotron radiation on the basis of Boltzmann's equation for photons which takes into account the influence of gravity to their motion and frequency. We arrive at the conclusion that the hypothesis of the existence in the Galaxy centre of the object with such extraordinary gravitational properties at least does not contradict observations.

Key words: dense matter, black hole physics, gravitation, accretion, radiation mechanism: non-thermal

1. Introduction

An analysis of stars motion in the Galaxy centre gives strong evidence for the existence here of a compact object with a mass of $(3 \div 4) 10^6 M_\odot$ associated with the radio source Sgr A* (Genzel et al. 2003a; Schödel et al. 2003; Ghez et al. 2003; Ghez et al. 2005). There are three kind of an explanation of observed peculiarities of the object emission:

- gas accretion onto the central object - a supermassive black hole (SMBH) (Coker & Melia 2000; Narayan & Yi 1995),
- ejection of magnetised plasma from an environment of the Schwarzschild radius of the SMBH (Falcke & Markoff 2000; Melia & Falcke 2001),
- explanations, based on hypotheses of other nature of the central object (clusters of dark objects (Maos 1998), a fermion ball (Viollier 2003), boson stars (Torres et al. 2000; Yuan, Narayan & Rees 2004).

In the present paper some consequences of the assumption that emission of Sgr A* caused by the existence in the Galaxy centre of a supermassive compact object without event horizon are examined. The existence of such stable configurations of the degenerated Fermi-gas of masses $10^2 \div 10^{10} M_\odot$, radiuses of which are smaller than the Schwarzschild radius r_g , is one of consequences (Verozub 1996) of the metric-

field equations of gravitation (Verozub 1991; Verozub 2001). According to the physical principles founding the equations, gravity of a compact object manifests itself as a field in Minkowski space-time for a remote observer in an inertial frame of reference, but become apparent as a space-time curvature for the observer in comoving reference frames associated with particles freely moving in this field. If the distance from a point attractive mass is many larger than r_g , the physical consequences, resulting from these equations, are very close to General Relativity results. However, they are principally other at short distances from the central mass. The spherically-symmetric solution of these equations in Minkowski space-time have no event horizon and physical singularity in the centre.

Since these gravitational equations were successfully tested by post-Newtonian effects in the solar system and by the binary pulsar PSR 1913+16 (Verozub & Kochetov 2000), and the stability of the supermassive configurations predicted by them was sufficiently strictly substantiated (Verozub & Kochetov 2001a), it is of interest to investigate the possibility of the existence of such a kind of an object in the Galaxy centre as an alternative to the hypothesis of a supermassive black hole.

The gravitational force of a point mass M affecting a free-falling particle of mass m is given by (Verozub 1991).

$$F = -m \left[c^2 C' / 2A + (A' / 2A - 2C' / 2C) \dot{r}^2 \right], \quad (1)$$

where

$$A = f'^2/C, \quad C = 1 - r_g/f, \quad f = (r_g^3 + r^3)^{1/3}. \quad (2)$$

In this equation r is the radial distance from the centre, $r_g = 2GM/c^2$, G is the gravitational constant, c is the speed of light at infinity, the prime denotes the derivative with respect to r .

For particles at rest ($\dot{r} = 0$)

$$F = -\frac{GmM}{r^2} \left[1 - \frac{r_g}{(r^3 + r_g^3)^{1/3}} \right] \quad (3)$$

Fig. 1 shows the force F affecting particles at rest and particles, free falling from infinity with zero initial speed, as the function of the distance $\bar{r} = r/r_g$ from the centre.

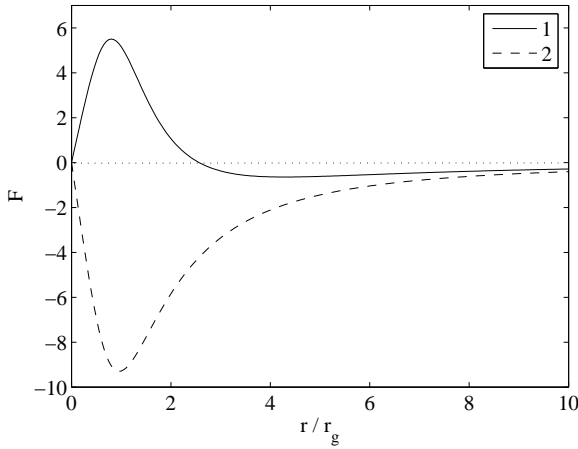


Fig. 1. The gravitational force (arbitrary units) affecting free-falling particles (curve 1) and particles at rest (curve 2) near a point attractive mass M .

It follows from Fig. 1 that the gravitational force affecting free falling particles changes its sign at $r \approx 2r_g$. Although we still never observed particles motion at distances of the order of r_g , we can verify this result for very remote objects in the Universe – at large cosmological redshifts. Indeed, consider a simple model - homogeneous selfgravitating dust-balls of different-size radii r_b . The force acting on particles at the surface of such a ball is given by Eq. (1) where, in this case, $r_g = (8/3)\pi c^{-2} G \varrho_b r_b^3$ is the Schwarzschild radius of the ball and ϱ_b is its density. If the ball density is of the order of the observed density of matter in the Universe ($\sim 10^{-29} \text{ g cm}^{-3}$), and the its radius r_b is not less than the radius of the observed matter ($\sim 10^{28} \text{ cm}$), then $r_b \leq r_g$. It follows from Fig. 1 that under the circumstances the radial accelerations F/m of speckle particles on the ball surface are positive. The repulsive force give rise a deceleration of the expansion of such a selfgravitating dust-like mass (Verozub 2002). More accurate calculations at the redshift $0 < z \leq 1$ (Verozub & Kochetov 2001b) give a good accordance with observation data (Riess et al. 1998).

We assume here for calculations that the mass M of the central object is about $3 \cdot 10^6 M_\odot$. The radius R of the object can be found from the distribution of the density ρ_m of matter inside the object. It can be obtained by solving the equations of the hydrodynamical equilibrium:

$$dp_m/dr = \rho_m g_m, \quad dM_r/dr = 4\pi r^2 \rho_m, \quad (4)$$

$$p_m = p_m(\rho_m).$$

In these equations M_r is the mass of matter inside the sphere \mathcal{O}_r of the radius r , $g_m = F/m$ is the force (3) per unit of mass, $r_g = 2G M_r/c^2$ is the Schwarzschild radius of the sphere \mathcal{O}_r and $p = p(\rho_m)$ is the equation of state of matter inside the object. The equation of state, which is valid for the density range of $(8 \div 10^{16}) \text{ g cm}^{-3}$, is given by (Harrison et al. 1965)

$$p_m = \left(\frac{n_m}{\partial n_m / \partial \rho_m} - \rho_m \right) c^2, \quad (5)$$

where in CGS units

$$n_m = Q_1 \rho_m \left(1 + Q_2 \rho_m^{9/16} \right)^{-4/9}, \quad (6)$$

$Q_1 = 6.0228 \cdot 10^{23}$ and $Q_2 = 7.7483 \cdot 10^{-10}$. For the gravitational force under consideration there are two types of the solutions of Eqs. (4). Besides the solutions describing white dwarfs and newtron stars, there are solutions with very large masses. For the mass of $3 \cdot 10^6 M_\odot$ the object radius resulting from Eqs. (4) is about $0.4 R_\odot = 0.034 r_g = 3 \cdot 10^{10} \text{ cm}$ where r_g is the Schwarzschild radius of the object.

It seems at first glance that accretion onto the object with a solid surface must lead to too large energy release which contradicts a comparatively low bolometric luminosity ($\sim 10^{36} \text{ erg s}^{-1}$) of Sgr A*. However, it must be taken into account that the radial velocity of test particles free falling from infinity is given by the equation (Verozub 1991)

$$v_{\text{ff}} = c \left[\frac{C}{A} (1 - C) \right]^{1/2}. \quad (7)$$

The velocity fast decreases at $r < r_g$. As a result, the velocity at the surface is only about $3.3 \cdot 10^8 \text{ cm s}^{-1}$. Consequently, at the accretion rate $\dot{M} = 10^{-7} M_\odot \text{ yr}^{-1}$ the amount of the released energy $\dot{M} v^2/2$ is equal to $3.4 \cdot 10^{35} \text{ erg s}^{-1}$.

2. Atmosphere

There are reasons to believe that the rate of gas accretion onto the supermassive object in the Galaxy centre due to star winds from surrounding young stars is of the order of $10^{-7} M_\odot \text{ yr}^{-1}$ (Coker 2001). Therefore, if the object has a solid surface, it can have an atmosphere, basically hydrogen. During 10 Myr (an estimated lifetime of surrounding stars) the mass M_{atm} of the gaseous envelope can reach $10 M_\odot$. If the density of the atmosphere is ρ_a , and the pressure is $p_a = 2 \rho_a k T / m_p$, where k is the Boltzmann constant, T is the absolute temperature and m_p is the proton mass, then the height of the homogeneous atmosphere is

$$h_a = 2 p_a / \rho_a g_m(R) = k T / m_p g_m(R), \quad (8)$$

where $g_m(R)$ is g_m at $r = R$. At the temperature $T = 10^7$ K the atmosphere height $h_a \sim 10^8$ cm. The atmospheric density $\rho_a = M_{\text{atm}}/4\pi R^2 h_a \sim 10^3 \text{ g cm}^{-3}$. Under such a condition, a hydrogen combustion must begin already in our time.

Of course, the accretion rate in the past could be of many orders more if surrounding stars have been born in a molecular cloud which was located in this region. (See a discussion in (Genzel et al. 2003a; Ghez et al. 2005)). In this case the combustion could begin many time ago and must be more intensive.

A relationship between the temperature and density can be found from the thermodynamical equilibrium equations

$$\frac{1}{4\pi r^2 \rho_a} \frac{dL_r}{dr} = \epsilon_{\text{pp}} + \lambda \frac{\dot{M} c^2}{M_{\text{atm}}} - T \frac{dS}{dt}, \quad (9)$$

where

$$L_r = 4\pi r^2 \frac{a c}{3\kappa \rho_a} \frac{d}{dr} T^4. \quad (10)$$

In these equations \dot{M} is the rate of the gas accretion, ϵ_{pp} is the rate of the nuclear energy generation per unit of mass, λ is the portion of the energy thermalized at accretion, S is entropy, and a is the radiation constant. For the proton-proton cycle the value of ϵ_{pp} can be taken to a first approximation as (Schwarzschild 1977; Bisnovatyi-Kogan 2001)

$$\epsilon_{\text{pp}} = 2.5 \cdot 10^4 X_{\text{H}}^2 \left(\frac{T}{10^9} \right)^{-2/3} \times \exp \left[-3.38 \left(\frac{T}{10^9} \right)^{-1/3} \right] \text{ erg g}^{-1} \text{ s}^{-1}, \quad (11)$$

where X_{H} is the hydrogen mass fraction which is assumed to be equal to 0.7, and

$$\kappa = 0.2(1 + X_{\text{H}}) + 4 \cdot 10^{24} (1 + X_{\text{H}}) \rho_a T^{-3.5} \text{ cm}^2 \text{ g}^{-1} \quad (12)$$

is the absorption coefficient caused by the Thomson scattering and free-free transitions.

In the stationary case for the homogeneous atmosphere we have

$$\frac{acT^4}{3\kappa\rho_a^2 h_a^2} = \epsilon_{\text{pp}} + \lambda \frac{\dot{M} c^2}{M_{\text{atm}}}. \quad (13)$$

Fig. 2 shows the relationship between T and ρ_a for several values of the parameter λ . It follows from the figure that the hydrogen burning can occur at the density of $10^2 \div 10^3 \text{ g cm}^{-3}$.

A luminosity from the burning layer and density distribution can be found by solving the differential equations system

$$\begin{aligned} \frac{dM_{\text{ra}}}{dr} &= 4\pi r^2 \rho_a, \quad \frac{dp_a}{dr} = \rho_a g(R), \\ \frac{dL_r}{dr} &= 4\pi r^2 \rho_a \epsilon_{\text{pp}}, \quad \frac{dT}{dr} = -\frac{3\kappa \rho_a L_r}{16\pi a c r^2 T^3}, \end{aligned} \quad (14)$$

where M_{ra} is the mass of the spherical layer from the surface to the distance r from the centre the object. The boundary conditions on the surface are of the form: $M_{\text{ra}}(R) = 0$,

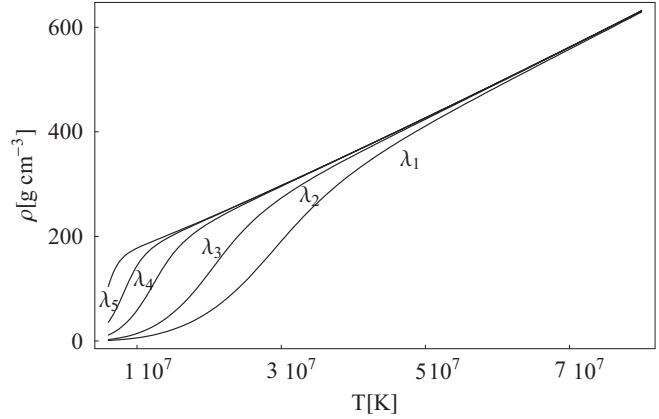


Fig. 2. Relationship between temperature and pressure of the homogeneous atmosphere for the values of the parameter λ : $\lambda_1 = 1$, $\lambda_2 = 1/6$, $\lambda_3 = 10^{-2}$, $\lambda_4 = 10^{-3}$, $\lambda_5 = 10^{-4}$ at $M = 3 \cdot 10^6 M_{\odot}$ and $\dot{M} = 10^{-7} M_{\odot} \text{ yr}^{-1}$

$\rho_a(R) = \rho_0$, $T(R) = T_0$, $L_r(R) = 0$. At $T_0 = 10^7$ K the luminosity $L = 2.4 \cdot 10^{31} \text{ erg s}^{-1}$ at $\rho_0 = 10^2 \text{ g cm}^{-3}$ and $L = 5.6 \cdot 10^{33} \text{ erg s}^{-1}$ at $\rho_0 = 10^3 \text{ g cm}^{-3}$.

The temperature at the surface cannot be much more than the above magnitude. At the temperature $T_0 = 3 \cdot 10^7$ K and $\rho_0 = 10^3 \text{ g cm}^{-3}$ the luminosity $L = 1.0 \cdot 10^{36} \text{ erg s}^{-1}$, i.e. is a value of the order of the observed bolometric luminosity of Sgr A*.

Due to gravitational redshift a local frequency (as measured by a local observer) differs from the one for a remote observer by the factor ¹

$$(1 + z_g)^{-1} = \sqrt{C}, \quad (15)$$

where the function $C = C(r)$ is defined by Eqs. (2). Fig. 3 shows the dependency of the redshift factor on the distance d from the surface of the object.

The difference between the local and observed frequency can be significance for the emission emerging from small distances from the surface. For example, at the above luminosity $L = 5.6 \cdot 10^{33} \text{ erg s}^{-1}$ the local frequency of the maximum of the blackbody emission is equal to $5.7 \cdot 10^{14} \text{ Hz}$. The observed frequency of the maximum is equal to $2.7 \cdot 10^{12} \text{ Hz}$. The corresponding specific luminosity is $\sim 2 \cdot 10^{21} \text{ erg s}^{-1} \text{ Hz}^{-1}$. At $L = 10^{36} \text{ erg s}^{-1}$ the frequency

¹ The simplest way to show it is follows. From the viewpoint of a remote observer, the energy integral of the motion of a particle in spherically-symmetric field is (Verozub 1991)

$$\dot{r} \frac{\partial \mathcal{L}}{\partial \dot{r}} + \dot{\varphi} \frac{\partial \mathcal{L}}{\partial \dot{\varphi}} - \mathcal{L} = E$$

where \mathcal{L} is the Lagrange function

$$\mathcal{L} = -mc(c^2 C - A\dot{r}^2 - B\dot{\varphi}^2)^{1/2}$$

and E is the energy of the particle. In the proper frame of reference of the particle the above equations take the form: $-\mathcal{L} = E$, and $\mathcal{L} = -mc^2 \sqrt{C}$. Photons can be considered as a particles of the effective proper mass $m_{\text{eff}} = h\nu/c^2$. It yields the relationship $\nu\sqrt{C} = \nu_{\infty}$, where ν_{∞} is the frequency at infinity.

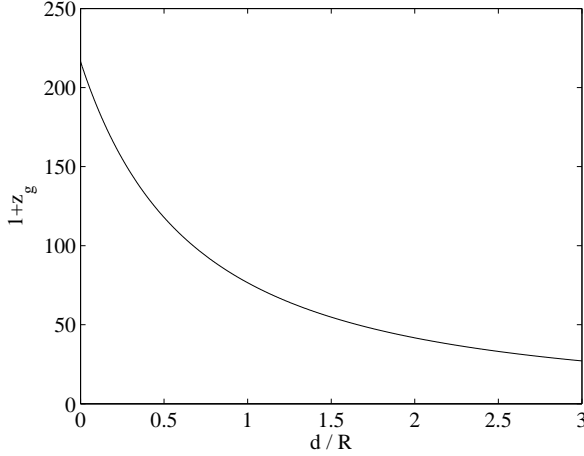


Fig. 3. The gravitational redshift close to the object surface of the mass $M = 3 \cdot 10^6 M_\odot$ and radius $R = 0.04 r_g$ as the function of the distance d from the surface (in units of R).

of the maximum is equal to $2 \cdot 10^{15}$ Hz , the observed frequency is equal to $9.7 \cdot 10^{12}$ Hz, and the specific luminosity is $\sim 1 \cdot 10^{23} \text{ erg s}^{-1} \text{ Hz}^{-1}$. These magnitudes are close to observation data. (Melia & Falcke 2001; Falcke et al. 2000)

3. Peculiarity of accretion

The main peculiarity of a spherical supersonic accretion onto supermassive objects without event horizon is the existence of the second sonic point – in a vicinity of the object which is not connected with hydrodynamical effects. The physical reason is that as the sound velocity v_s in the incident flow grows together with the temperature, the gas velocity v , like the velocity of free falling particles (7) , begin to decrease close to $r = r_g$. As a result, the equality $v = v_s$ take place at some distance $r_s > R$, not far from the surface of the object.

According to the used gravitation equations (Verozub 1991), the maximal radial velocity of a free falling particle does not exceed $0.4 c$. Therefore, the Lorentz-factor is nearly 1, and to find r_s we can proceed from the simple hydrodynamics equations:

$$4\pi r^2 v = \dot{M}, \quad vv + n'/n = g \quad (16)$$

$$\left(\frac{\varepsilon}{n}\right)' - P \frac{n'}{n} = 0.$$

In these equations, n is the particles number density, ε is the energy density of the gas, $g = F/m$ is the gravitational acceleration. The gas pressure P is taken as $2nkT$, and the prime denotes the derivative with respect to r . Therefore, as in the case of the Bondi accretion model, we neglect radiation pressure , an equipartition magnetic field which may exist in the accretion flow, viscosity and loss of energy due to radiation. However, the energy density of the infalling gas in (16) is calculated more accurate (Coker & Melia 2000):

$$\varepsilon = m_p c^2 n + \phi n k T, \quad (17)$$

where

$$\phi = 3 + x \left(\frac{3K_3(x) + K_1(x)}{4K_2(x)} - 1 \right) + y \left(\frac{3K_3(y) + K_1(y)}{4K_2(y)} - 1 \right), \quad (18)$$

$x = m_e c^2 / kT$, $y = m_p c^2 / kT$ and K_j ($j = 2, 3$) is the i^{th} order modified Bessel function, m_e is the electron mass

The sound velocity v_s as a function of r can be found (Service 1986) as

$$v_s = c \left(\frac{\Gamma P}{\rho c^2 + P} \right)^{1/2}, \quad (19)$$

where $\rho = m_p n$ is the gas (fully ionised hydrogen plasma) density. The adiabatic index

$$\Gamma = c_p / c_v, \quad (20)$$

where

$$c_p = 5 \varsigma \Theta^{-1} - (\varsigma^2 - 1) \Theta^2, \\ c_v = c_p - 1, \quad (21) \\ \varsigma = K_3(\Theta^{-1}) / K_2(\Theta^{-1}),$$

and $\Theta = kT / m_e c^2$. Then

$$\frac{P}{\rho c^2 + P} = \frac{\Theta}{\eta}, \quad (22)$$

where $\eta = K_3(\Theta^{-1}) / K_2(\Theta^{-1})$. Having used results by Press et al. 1986 Service (Service 1986) have obtained the following polynomial fitting for Γ and $P/(\rho c^2 + P)$:

$$P/(\rho c^2 + P) = 0.03600y + 0.0584y^2 - 0.1684y^3, \\ \Gamma = (5 - 1.8082y + 0.8694y^2 - 0.3049y^3 + 0.2437y^4), \quad (23)$$

where $y = \Theta / (0.36 + \Theta)$. We assume that $\dot{M} = 10^{-7} M_\odot \text{ yr}^{-1}$ and that at the distance from the centre $r_0 = 10^{17} \text{ cm}$ the velocity $v(r_0) = 10^8 \text{ cm s}^{-1}$ and the temperature $T(r_0) = 10^4 \text{ K}$. Under these conditions $r_s = 8R$. At this point the solutions of the equations have a peculiarity. The temperature $T(r_s) \sim 10^{10} \text{ K}$. The value of r_s weakly depends on \dot{M} , however, it is more sensitive to physical conditions at large r . For example, it varies from the above magnitude $8R$ up to $22R$ at the temperature $T(r_0) = 10^6 \text{ K}$.

The postshock region stretches from r_s to such a depth where protons stopping occurs due to Coulomb collisions with atmospheric electrons. (We do not take into account collective effects in plasma). The change in the velocity v_p of infalling protons in plasma is given by the solution of the differential equation

$$v_p \frac{dv_p}{dr} = g + w, \quad (24)$$

where w is the deceleration of protons due to Coulomb collisions. This magnitude is of the form (Alme & Wilson 1973; Li & Petrasso 1993; Deufel, Dullemond & Spruit 2001)

$$w = -f(x_e) \frac{4\pi N e^4}{m_e m_p v_p} \ln \Lambda. \quad (25)$$

In this equation e is the electron charge, N is the particles number density. At $r < 1.3R$ the last magnitude is taken here as the quantity $n_a = \rho_a/m_a$ resulting from the solution of Eqs. (14), and as the particles number density in a free-falling gas

$$n = \frac{\dot{M}}{4\pi r^2 v_{ff} m_p}, \quad (26)$$

at $r > 1.3R$, where $n > n_a$. The magnitude

$$\ln \Lambda = \ln \frac{3}{2\sqrt{\pi} e^3} \frac{(kT)^{3/2}}{N^{1/2}} \quad (27)$$

is the Coulomb logarithm. The function $f(x_e)$ is taken here as

$$f(x_e) = \psi(x_e) - x_e(1 + m_e/m_p)\psi'(x_e), \quad (28)$$

where

$$x_e = \left(\frac{m_e v_p^2}{2kT} \right)^{1/2}, \quad (29)$$

ψ is the error function

$$\psi(x_e) = \frac{2}{\sqrt{\pi}} \int_0^{x_e} \exp(-z^2) dz \quad (30)$$

and $\psi'(x_e)$ is the derivative with respect to r .

The velocity v_{ff} is close to the greatest possible velocity of the infalling gas. It leads to the minimally possible distance of a point of the full proton stopping from the object surface. A numerical solution of Eq. (24) shows that this magnitude is equal to $(0.3 \div 0.4)R$. The particles number density in this region is $\sim 10^{14} \text{ cm}^{-3}$ (We neglect some difference between the height of protons and electrons stopping (Bildsten et al. 1992)). Above this region the most part of the kinetic energy protons is released.

The distribution of the temperature $T(r)$ in this region can be obtained from an equation of the energy balance. (Such a question for neutron stars has been considered earlier by Zel'dovich & Shakura (1969) , Alme & Wilson (1973), Turolla et al. (1994), Deufel et al. (2001), and, for magnetic white dwarfs, by Woelk & Beuermann (1992)). Suppose that all protons stopping power is converted into radiation. Let H be the radiative flux and $L_r = 4\pi r^2 H$ – the flux trough the sphere of the radius r . Then the energy balance means that

$$L_r' = 4\pi r^2 \epsilon_{cul} \rho \quad (31)$$

where ϵ_{cul} is the rate of the energy release due to Coulomb collisions, $\rho = m_p N$ is the gas density and the prime denotes the derivative with respect to r . The left-hand side in (31) is

$$L_r' = 4\pi r^2 \left(H' + \frac{2}{r} H \right). \quad (32)$$

The first-order moment equation of the standard transfer equation for a spherically- symmetric gray atmosphere can be written as (Schwarzschild 1977)

$$H' + \frac{2}{r} H + c\kappa\rho U - j\rho = 0, \quad (33)$$

where U is the density of the radiation energy, j and κ are the mean emission and adsorbition coefficients, respectively.

Eqs. (32) and (33) yield

$$L_r' 4\pi r^2 \rho = (j - c\kappa U). \quad (34)$$

Therefore, the appropriate for the purpose of this paper equation of the energy balance is of the form

$$\epsilon_{cul} = j - c\kappa U. \quad (35)$$

To a first approximation, j and κ can be taken as (Bisnovatyi-Kogan 2001)

$$j = 5 \cdot 10^{20} \rho T^{1/2} + 6.5 U T + 2.3 T B^2 \text{ erg g}^{-1} \text{ s}^{-1} \quad (36)$$

and

$$\kappa = 0.4 + 6.4 \cdot 10^{22} \rho T^{-7/2} + 6.5 c^{-1} T_\gamma + 2 \cdot 10^2 B^2 T^{-3} \text{ cm}^2 \text{ g}^{-1}, \quad (37)$$

where $T_\gamma = (U/a)^{1/4}$ is the radiation temperature and B is a possible magnetic field close to the object surface (See Section 5). Terms in (36) describe free-free, Compton and synchrotron emission, respectively. Terms in (37) describe the inverse processes, and the first term takes into account the Thomson scattering.

The rate ϵ_{cul} of the energy release due to Coulomb collisions per unit of mass is given (Alme & Wilson 1973) by

$$\epsilon_{cul} = f(x_e) \frac{4\pi e^4}{m_p v_p} \ln \Lambda. \quad (38)$$

If to neglect adsorbition terms in Eq. (35) and the gravitation influence inside the stopping region , it is possible to obtain some simple analytic relationships between the temperature T and the energy E_p of a decelerating photon. Indeed, the value of N at the distance $r = 6R$ is of the order of 10^8 cm^{-3} . At the temperatures $\sim 10^{10} \text{ K}$ the free-free and Compton radiations are predominated. Taking into account that $x_e \ll 1$, we obtain

$$f(x_e) \approx \frac{4}{3\sqrt{\pi}} \left(\frac{m_e}{m_p} \right)^{3/2} \left(\frac{E_p}{kT} \right)^{3/2}, \quad (39)$$

which yields the rate of the energy release per unit of mass:

$$\epsilon_{cul} = \xi \frac{N E_p}{T^{3/2}}, \quad (40)$$

where

$$\xi = \frac{16\sqrt{\pi} m_e e^4}{3\sqrt{2} m_p^2 k^{3/2}}. \quad (41)$$

Then, if the largest contribution provides free-free radiation (the field B is weak), it follows from the equation of the energy balance that

$$T = (\delta_1 E_p)^{1/2} \quad (42)$$

where $\delta_1 = 2.36 \cdot 10^{24} \text{ K}^2 \text{ erg}^{-1}$. For example, at $r = 6R$ the proton energy $E_p = 4.8 \cdot 10^{-6} \text{ erg}$ which yields the temperature $T = 3.3 \cdot 10^9 \text{ K}$. Since E_p is a fast decreasing function close to $r = 1.4R$, a sharp drop in the temperature takes place at the distance $\sim 0.4R$ from the centre.

If the magnetic field is sufficiently strong, the main contribution in the energy balance provides synchrotron radiation. In this case the relationship between the temperature T and the energy E_p of the proton takes the form

$$T = \left(\delta_2 \frac{N E_p}{B^2} \right)^{2/5} \quad (43)$$

where $\delta_2 = 10^{24} \text{ K}^{5/2} \text{ erg}^{-1} \text{ cm}^3$ and $B \neq 0$.

In general case, assuming that at the distance $r_0 = 6R$ the luminosity $L(r_0) = 10^{36} \text{ erg s}^{-1}$, the energy flux $U(r_0) = L(r_0)/4\pi r_0^2 c$ and the protons velocity $v_p(r_0) = v_H(r_0)$, we find from Eq. (35) that the temperature $T(r_0) = 5.4 \cdot 10^9 \text{ K}$.

On the other hand, if the absorption is not ignored, Eq. (35) yields the differential equation

$$T' = - \frac{\partial \Phi(r, T) / \partial r}{\partial \Phi(r, T) / \partial T}, \quad (44)$$

where $\Phi(r, T) = \epsilon_{\text{cul}} - j + c\kappa U$. A simultaneously solving of this differential equation together with the equations

$$\frac{1}{3} U' = \kappa \frac{L_r}{4\pi r^2 c}$$

and (31) allows to obtain the distribution $T(r)$ at the distances r where $\epsilon_{\text{cul}} \neq 0$.

Fig. 4 shows a typical temperature profile in the region of protons stopping.

It is interesting to estimate the temperature at the surface proceeding from the simplifying assumption that all luminosity of Sgr A* ($L \sim 10^{36} \text{ erg s}^{-1}$) is caused by nuclear burning at the surface and stopping of incident protons at some distance $d \leq R$ from the surface. Under such a condition, an effective temperature T_0 on the top of the dense atmosphere ($d \sim 0.4R$) is about 10^4 K . The radial distribution of the temperature in the diffusion approximation can be determined by the differential equation

$$(T^4)' = \frac{3\chi}{4b_s} H, \quad (45)$$

where $\chi = \kappa\rho$, $b_s = 5.75 \cdot 10^{-5} \text{ erg cm}^{-2} \text{ K}^{-4}$ is the Stephan-Boltzmann constant, and H is the radiative flux which is supposed to constant. It gives approximately

$$T^4 - T_0^4 = \frac{\chi}{b} H \Delta r, \quad (46)$$

where $\Delta r = 0.4R$. Setting $\chi = 1$ we find that at the surface the temperature is about 10^7 K which coincides with the magnitude used in Section 2.

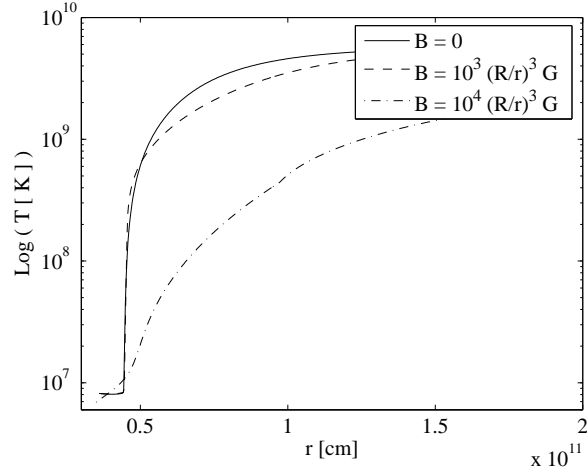


Fig. 4. The temperature T as the function of r inside the region of protons stopping for the object of the radius $R = 0.04r_g$. The boundary conditions are: at the distance from the surface $5R$ the luminosity $L_0 = 10^{36} \text{ erg s}^{-1}$, the temperature $T_0 = 5.4 \cdot 10^9 \text{ K}$, the velocity of protons $v_{p0} = 2.32 \cdot 10^9 \text{ cm s}^{-1}$. The plots for three value of the magnetic field B are shown.

To find the temperature distribution from the surface to distances $d \sim R$ more correctly, equations of the structure of lower atmosphere, accretion and energy balance should be solved simultaneously. It can affect the value of the atmospheric density at the surface. However, as it was noted in Section 2, our knowledge of this magnitude initially is rather uncertain. The density of $\rho \sim 10^3 \text{ g cm}^{-3}$ (which corresponds to the temperature $T \sim 10^7 \text{ K}$) is the most probable since an essentially greater value of the density or temperature of the bottom atmosphere leads to the luminosity which contradicts the observable bolometric luminosity of Sgr A*. At the same time the existence of a high temperature at distances where protons stopping occurs hardly is a subject to doubt. Therefore, the jump of the temperature of the order of $(10^2 \div 10^3) \text{ K}$ is an inevitable consequence of the Sgr A* model under consideration.

4. Comptonization

An emergent intensity I of the low-atmospheric emission after passage through a hot spherical homogeneous layer of gas is a convolution of the Plankian intensity I_0 of the emission of a burning layer, and the frequency redistribution function $\Phi(s)$ of $s = \lg(\nu/\nu_0)$, where ν_0 and ν are the frequencies of the incoming and emerging radiation, respectively:

$$I(x) = \int_{-\infty}^{\infty} \Phi(s) I_0(s) ds. \quad (47)$$

At the temperature of the order of 10^{10}K the dimensionless parameter $\Theta = kT/m_e c^2 \sim 1$. Under the condition the function $\Phi(s)$ can be calculated (Birkinshaw 1999) as follows:

$$\Phi(s) = \sum_{k=0}^m \frac{e^{-\tau} \tau^k}{k!} P_k(s), \quad (48)$$

where

$$P_1(s) = \int_{(\beta_e)_{\min}}^1 \varphi(\beta_e) P(s, \beta_e) d\beta_e. \quad (49)$$

In the last equations β_e is the electron-light velocity speed ratio,

$$\varphi(\beta_e) = \frac{\gamma^5 \beta_e^2 \exp(-\gamma/\Theta)}{\Theta K_2(\Theta^{-1})}, \quad (50)$$

$$\gamma = (1 - \beta_e^2)^{-1/2},$$

$$(\beta_e)_{\min} = \frac{e^{|\delta|} - 1}{e^{|\delta|} + 1} \quad (51)$$

and

$$P(s, \beta_e) = \frac{3}{16\gamma^4 \beta_e} \int_{\mu_1}^{\mu_2} (1 - \beta_e \mu)^{-3} \times \left(1 + \beta_e \mu' (1 + \mu^2 \mu'^2 + \frac{1}{2}(1 - \mu^2)(1 - \mu'^2)) \right) d\mu, \quad (52)$$

$$\mu' = \frac{e^s (1 - \beta_e \mu) - 1}{\beta_e},$$

$$\mu_1 = \begin{cases} -1 & s \leq 0 \\ \frac{1 - e^{-s}(1 + \beta_e)}{\beta_e} & s \geq 0 \end{cases} \quad (53)$$

$$\mu_2 = \begin{cases} \frac{1 - e^{-s}(1 - \beta_e)}{\beta_e} & s \leq 0 \\ 1 & s \geq 0. \end{cases} \quad (54)$$

Taking into account scattering up to 3rd order ($m = 3$) an approximate emergent spectrum of the nuclear radiation after comptonization has been obtained for the case when the temperature at the surface is 10^7K and $\rho = 3 \cdot 10^3 \text{gcm}^{-3}$. The results are plotted in Fig. 5 for several values of the optical thickness τ .

It follows from this figure that the comptonization may give a contribution to the observed Sgr A* radiation (Baganoff et al. 2003; Porquet et al. 2003) in the range $10^{15} \div 10^{20} \text{Hz}$.

It is necessary to note that the timescale Δt of a variable process is connected with the size Δ of its region by the relationship $\Delta \leq c \Delta t / (1 + z_g)$. For example, the variations in the radiation intensity of $\sim 600 \text{s}$ on clock of a remote observer occur at the distance $d \leq 1.7R$ from the surface. For this reason high-energy flashes can be interpreted as processes near the surface of the central objects.

It is possible that the diffuse, hard X-ray emission from the giant molecular clouds of central $100pc$ of our Galaxy (Park et al. 2004; Cramphorn & Sunyaev 2002) is a consequence of the more intensive X-ray emission of Sgr A* due to nuclear bursts in the past.

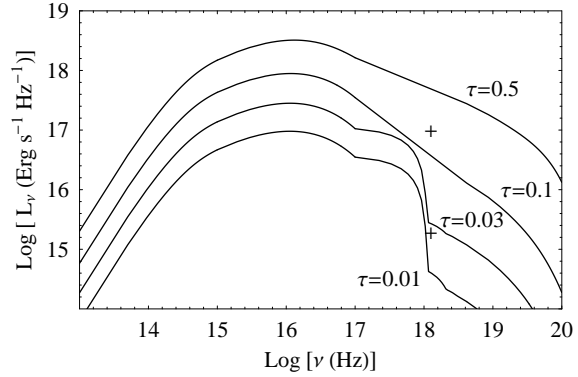


Fig. 5. Emergent spectra of nuclear burning after comptonization by a homogeneous hot layer for the optical thickness $\tau = 0.01, 0.03, 0.1, 0.5$. The crosses denote the observation data (Baganoff et al. 2003; Porquet et al. 2003) (The upper and lower limits are specified).

5. Transfer equation and synchrotron radiation of Sgr A*

Some authors (Robertson & Leiter 2002) find in spectra of candidates into Black Holes some evidences for the existence of a proper magnetic field of these objects that is incompatible with the existence of an event horizon. In order to show how the magnetic field near the surface of the object in question can manifests itself, we calculate the contribution of a magnetic field to the spectrum of the synchrotron radiation. We assume that the magnetic field is of the form

$$B_{\text{int}} = B_0 \left(\frac{R}{r} \right)^3, \quad (55)$$

where $B_0 = 1.5 \cdot 10^4 \text{G}$.

To find the emission spectrum from the surface vicinity, the influence of gravitation on the frequency of photons and their motion must be taken into account. An appropriate transfer equation is the Boltzmann equation for photons which uses equations of their motion in strong gravitational fields resulting from the used gravitation equations. (Such an equation in General Relativity has been considered by Lindquist (1966), Schmidt-Burgk (1978), Zane et al. (1996) and Papathanassiou & Psaltis (2000)) The Boltzmann equation is of the form

$$\frac{d\mathcal{F}}{dt} = St(\mathcal{F}, t, \mathbf{x}), \quad (56)$$

where $\mathcal{F} = \mathcal{F}(t, \mathbf{x}, \mathbf{p})$ is the phase-space distribution function, $\mathbf{x} = (x^1, x^2, x^3)$ and $\mathbf{p} = (p^1, p^2, p^3)$ are the photon coordinates and momentums, d/dt is the derivative along the photon path, $St(\mathcal{F}, t, \mathbf{x})$ is a collisions integral. The solution of this equation is related to the intensity I as $\mathcal{F} = \beta^{-1} I$ where $\beta = 2h^4 \nu^3 c^{-2}$ and ν is the local frequency.

In the stationary spherically-symmetric field the distribution function $\mathcal{F} = \mathcal{F}(r, \mu, \nu)$ where $\mu = \cos \theta$ is the cosine between the direction of the photon and the surface normal.

The path derivative takes the form

$$\frac{d\mathcal{F}}{dt} = v_{\text{ph}} \left(\frac{\partial \mathcal{F}}{\partial r} + \frac{\partial \mathcal{F}}{\partial \nu} \frac{\partial \nu}{\partial r} + \frac{\partial \mathcal{F}}{\partial \mu} \frac{\partial \mu}{\partial r} \right), \quad (57)$$

where the radial component of the photon velocity $dr/dt = v_{\text{ph}}$ is given by (Verozub 1996)

$$v_{\text{ph}} = c \left[\frac{C}{A} \left(1 - \frac{Cb^2}{f^2} \right) \right], \quad (58)$$

and b is an impact parameter. The frequency as a function of r is given by the equation $\nu = \sqrt{C} \nu_{\infty}$. The function $\mu = \mu(r)$ satisfy the differential equation

$$\frac{d\mu}{dr} = (1 - \mu^2)^2 \frac{d\theta}{dr}, \quad (59)$$

where (Verozub 1996)

$$\frac{d\theta}{dr} = \frac{cCb}{f^2}. \quad (60)$$

In the above equations the geometrical relationship $b^2 = r^2(1 - \mu^2)$ must be taken into account.

With regards to all stated above, the Boltzmann equation along the photon paths can be taken in the form

$$v_{\text{ph}} \frac{d\mathcal{F}}{dr} = \frac{1}{\beta} (\eta + \sigma J) - (\chi + \sigma) \mathcal{F}. \quad (61)$$

The right-hand side of this equation is the ordinary simplified collisions integral for an isotropic scattering (Mihalas 1980) in terms of the function \mathcal{F} . In this equation

$$J = (1/2) \int_{-1}^1 \beta \mathcal{F} d\mu, \quad (62)$$

is the mean intensity, η is the synchrotron emissivity, χ is the true adsorption opacity, σ is the Thomson scattering opacity.

If $\mathcal{F}_{\nu}(r, \mu)$ is a solution of the integro-differential equation (61), the specific radiative flux is

$$H_{\nu} = (1/2) \int_{-1}^1 \beta \mathcal{F}_{\nu}(r, \mu) \mu d\mu \quad (63)$$

Therefore, by solving Eq. (61) for all parameters $0 < b < \infty$ an emergent spectrum of Sgr A* can be obtain. For a numerical solving an iterative method can be used with no scattering as a starting point.

We assume that at the surface of the object (due to nonstationary MHD-processes) and also higher, in the shock region, a small fractions of the electrons may exist in a non-thermal distribution. Therefore, following Yuan, Quatert & Narayan (2004), we setting $\eta = \eta_{\text{th}} + \eta_{\text{nth}}$ and $\chi = \chi_{\text{th}} + \chi_{\text{nth}}$, where indexes th and nth refer to thermal and nonthermal population, respectively.

There is a fitting function for the synchrotron emissivity of the thermal distribution of electrons which is valid from $T \sim 10^8 \text{K}$ to relativistic regimes (Madehavan, Narayan & Yi 1996):

$$\eta_{\text{th}} = \frac{Ne^2\nu}{\sqrt{3}cK_2(1/\Theta)} \mathcal{M}(\zeta), \quad (64)$$

where, $K_2(1/\Theta)$ is the modified Bessel function of the second order,

$$\mathcal{M}(\zeta) = \frac{4q_1}{\zeta^{1/6}} \left(1 + \frac{0.4q_2}{\zeta^{1/4}} + \frac{0.53q_3}{\zeta^{1/2}} \right) e^{-1.8896\zeta^{1/3}}, \quad (65)$$

$\zeta = 2\nu/3\nu_b\Theta^2$ and $\nu_b = eB/2\pi m_e c$ is the nonrelativistic cyclotron frequency. The parameters q_1 , q_2 and q_3 are functions of the temperature T . They have been obtained from Table 1 of the above paper. The true adsorption opacity χ_{th} is supposed to be related to η_{th} via Kirchoff law as $\chi_{\text{th}} = \eta_{\text{th}}/B_{\nu}$, where B_{ν} is the Planck function.

For the population of non-thermal electrons we use a power-law distribution (Özel, Psaltis & Narayan 2000) which is proportional to $\gamma^{-\alpha}$ where γ is the Lorentz factor and the parameter α is supposed to be equal to 2.5. For such a distribution the emissivity of the non-thermal electrons is approximately given by

$$\eta_{\text{nth}} = 0.53 \delta \frac{e^2 N}{c} \varphi_1(\Theta) \nu_b \left(\frac{\nu}{\nu_b} \right)^{-0.25}, \quad (66)$$

where

$$\varphi_1(\Theta) = \Theta \frac{6 + 15\Theta}{4 + 5\Theta} \quad (67)$$

and the fraction δ of the electrons energy in the thermal distribution is supposed to be the constant in radius and equal to 0.05.

The adsorption coefficient for non-thermal electrons is taken as

$$\chi_{\text{nth}} = 1.37 \cdot 10^{27} \delta \frac{e^2 N}{c\nu} \varphi(\theta) \nu_b \left(\frac{\nu}{\nu_b} \right)^{2.75}. \quad (68)$$

For a correct solution of Eq. (61) is necessary to take into account that from the point of view of the used gravitational equations there are three types of photons trajectories in the spherically-symmetric gravitation field. It can be seen in Fig. 6. The figure shows the locus in which radial photon velocities are equal to zero. It is given by the equation

$$b = \frac{f}{\sqrt{C}}. \quad (69)$$

The minimal value of b is $b_{\text{cr}} = 3\sqrt{3}r_g/2$. The corresponding distance from the centre is $r_{\text{cr}} = \sqrt[3]{19}r_g/2$. The photons with an impact parameter $b > b_{\text{cr}}$, the trajectories of which start at $r < r_{\text{cr}}$ (type 1 in Fig. 6), cannot move to infinity. It can do photons with $b > b_{\text{cr}}$, if their trajectories start at $r > r_{\text{cr}}$ (type 3), and all photons with $b < b_{\text{cr}}$ (type 2).

In order to find the solutions \mathcal{F}_+ of equation (61), describing the outgoing emission provided by photons of type 2, the boundary condition $\mathcal{F}(r = R, \nu, \mu) = 0$ can be used. The boundary condition $\mathcal{F}(r \rightarrow \infty, \nu, \mu) = 0$ allows to find the solutions \mathcal{F}_- , describing the emission ingoing from infinity. The values of this solution can be used as a boundary conditions to find the outgoing emission provided by the photons of type 3 (Zane et al. 1996). Of course, the value of the function $\mathcal{F} = \beta^{-1}B_{\nu}$, where B_{ν} is the Planck function, can

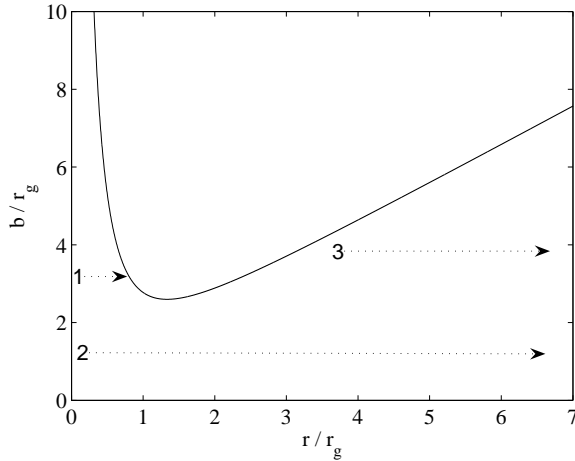


Fig. 6. The function $b(r)$. There are 3 types of photons trajectories: 1) The impact parameter $b > b_{cr}$, the trajectory begins at $r < r_{cr}$, 2) $b < b_{cr}$, 3) $b > b_{cr}$, the trajectory begins at $r > r_{cr}$.

be used as a boundary condition to find emission from large optical depths.

Fig. 7 shows the spectrum of the synchrotron radiation in the band of $10^{11} \div 2 \cdot 10^{18}$ Hz for three cases:

- for the magnetic field B_{int} (55) of the object at the gibrid distribution of electrons with the parameter $\delta = 0.05$ (the dashed line),
- for the sum of B_{int} and an external equipartition magnetic field $B_{ext} = (\dot{M} v_{ff} r^{-2})^{1/2}$ which may exist in the accretion flow (Coker & Melia 2000) (the solid line),
- for the magnetic field B_{int} without non-themal electrons (the dotted line which at the frequencies $< 10^{14}$ Hz coincides with the dash line). in the presence of the proper magnetic field

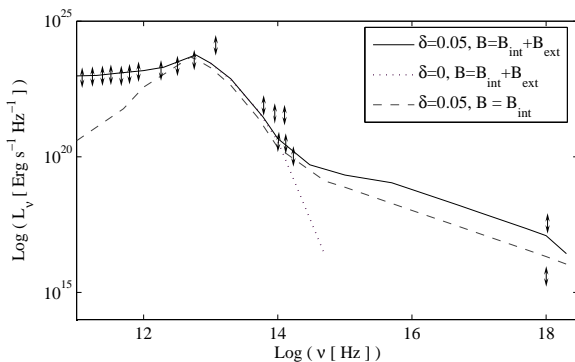


Fig. 7. The spectrum of the synchrotron radiation. The dashed line shows the luminosity L_ν for a remote observer for the possible proper magnetic field B_{int} . The parameter δ of the hybrid distribution of electrons is equal to 0.05. The dotted line shows the luminosity without non-themal electrons. The solid line is the luminosity at the presence both B_{int} and the external equipartition magnetic field B_{ext} . The short lines with the double arrows show observation data according to Özel et al (2000)

It follows from the figure that the proper magnetic field can manifest itself for the remote observer at the frequencies larger that $(2 \div 4) \cdot 10^{12}$ Hz – in IR and X-ray radiation.

It follows from Figs. 5 and 7 that the peculiarity of Sgr A* spectrum at $\nu > 10^{14}$ Hz can be explained by both physical processes at the object surface and by the existence of a non-thermal fraction of electrons in the gas environment. We do not know at the moment which of these sources is predominant. However, the opportunity of the explanation of flare activity of Sgr A* by physical processes at the surface is rather attractive.

The measurements of the polarisation of Sgr A* emission offer new possibilities for testing of models of this object. According to observations by Bower et al. (2003) at the frequency $2.3 \cdot 10^{11}$ Hz, the Faraday rotation measure (RM) is $\leq 2 \cdot 10^6 \text{ rad m}^{-2}$. In our case at this frequency the optical depth τ of the atmosphere become equal to 1 at a distance of $r_{min} \approx 7 r_g$ from the centre. The influence of the internal magnetic field is insignificant at here. The value of the RM is $\sim 10^7 \text{ rad m}^{-2}$. This magnitude is not differed from that obtained in the model by Yan, Eliot & Narayan (2004) based on the SMBH - theory. Taking into account that we know the distribution of the density and magnetic field very approximately, we cannot say that the obtained value of the RM contradict observation data.

At increasing of the frequency the value of r_{min} decreases, and influence of the proper magnetic field must increase because the radiation come from more depth of the atmosphere. At frequencies more than $2 \cdot 10^{12}$ Hz the atmosphere become the transparent completely to the synchrotron radiation. Therefore, the full depolarization of the radiation may give some evidence for the existence of a magnetic field close the surface.

6. Conclusion

The clarification of nature of compact objects in the galactic centres is one of actual problems of fundamental physics and astrophysics. The results obtained above, of course, yet do not allow to draw the well-defined conclusions. However they show that the possibility, investigated here, at last, does not contradict the existing observations, and require further study.

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